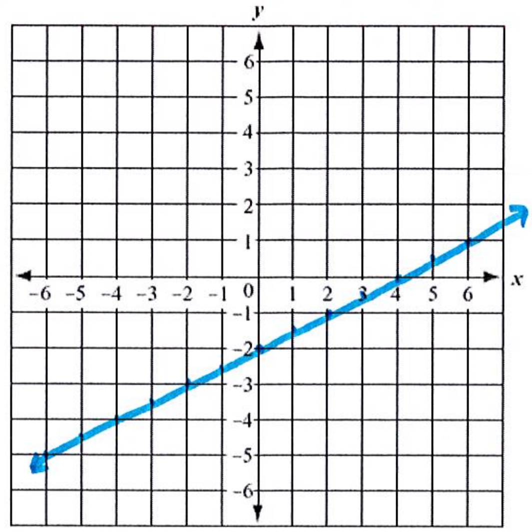
Mr. Michael T. Davis Calculus	Section 3.1 & 3.2 Practice Quiz December 5, 2016

Name: Solutions

1. Sketch the graph of a continuous function $y = f(x)$ that has the following properties:

   i. $f(0) = -2$,

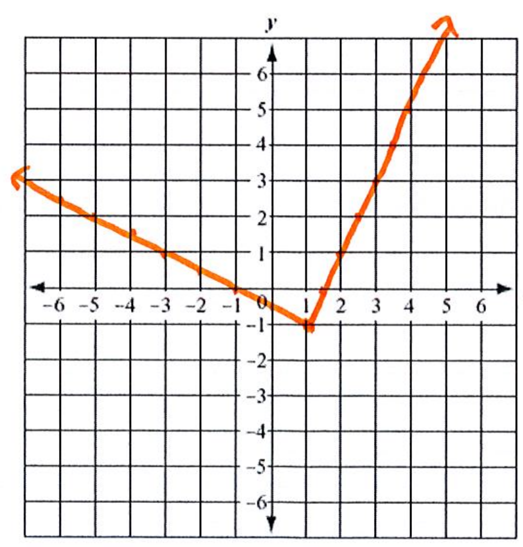
   ii. $f'(x) = \frac{1}{2}$.



2. Sketch the graph of a continuous function $y = f(x)$ that has the following properties:

   i. $f(1) = -1$,

   ii. $f'(x) = \begin{cases} \frac{-1}{2} \text{ if } x < 1 \\ 2 \text{ if } x \geq 1 \end{cases}$.



3. Write an equation for a linear function $y = f(x)$ that has the following properties:

   i. $f(-1) = 3$,

   ii. $f'(-1) = 2$.

$y - 3 = 2(x - {}^{-}1)$

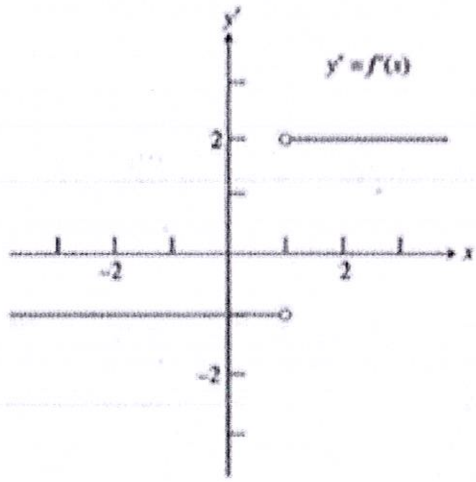
$y - 3 = 2(x + 1)$

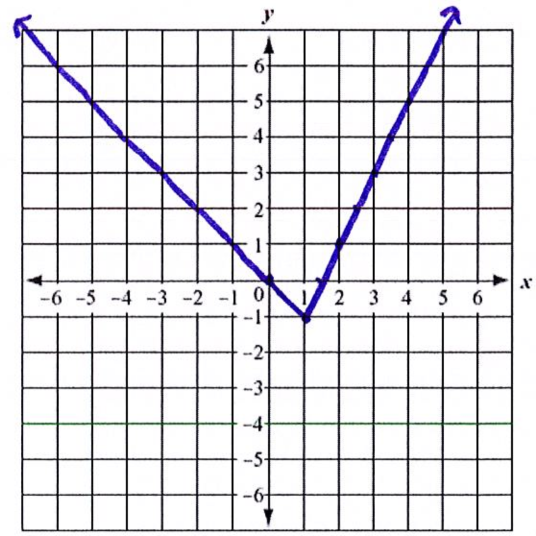
4. Sketch the graph of a function $y = f(x)$ that has the following properties:

i. $f(0) = 0$,

ii. The graph of f', the derivative of f, is as shown in the figure on the left below,

iii. f is continuous for all x.





5. Show that the function $f(x) = \begin{cases} x^2 \text{ if } x \le 0 \\ 2x \text{ if } x > 0 \end{cases}$ has left-hand and right-hand derivatives at $x = 0$, but no derivative at $x = 0$. You may use the derivative rules here.

$f'(x) = \begin{cases} 2x \text{ if } x \le 0 \\ 2 \text{ if } x > 0 \end{cases}$ left hand derivative is $2(0) = 0$
right hand derivative is 2

Since the left hand derivative is not equal to the right derivative, the function is not differentiable at $x=0$.

6. Use the definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find the derivative of the function $f(x) = \frac{2}{x}$ at $a = 3$. You may not use the derivative rules here.

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{\frac{2}{3+h} - \frac{2}{3}}{h} = \lim_{h \to 0} \frac{\frac{6}{9+3h} - \frac{6+2h}{9+3h}}{h}$$

$$= \lim_{h \to 0} \left(\frac{6-6-2h}{9+3h} \cdot \frac{1}{h} \right) = \lim_{h \to 0} \left(\frac{-2h}{9+3h} \cdot \frac{1}{h} \right) = \lim_{h \to 0} \frac{-2}{9+3h} =$$

$$= \frac{-2}{9}$$

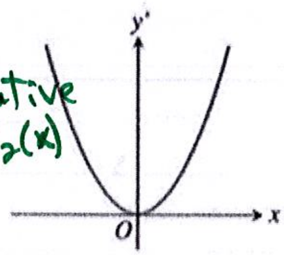
7. Use the definition $f'(a)=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$ to find the derivative of the function $f(x)=\sqrt{x}$ at $a=9$. You may not use the derivative rules here.

$$f'(9)=\lim_{x\to 9}\frac{f(x)-f(9)}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-\sqrt{9}}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$$

$$=\lim_{x\to 9}\frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)}=\lim_{x\to 9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}$$

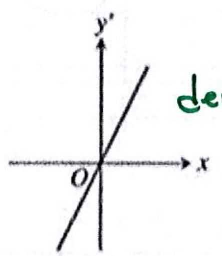
8. Match the graph of each function shown at the bottom with its derivative function shown immediately below.

derivative of $f_2(x)$



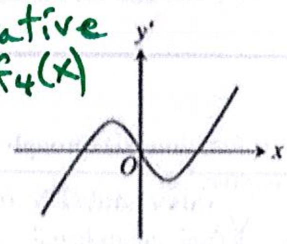
(a)

derivative of $f_1(x)$



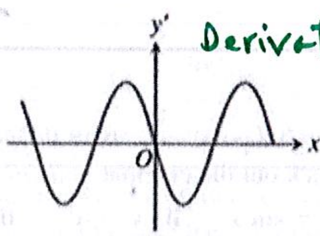
(b)

derivative of $f_4(x)$



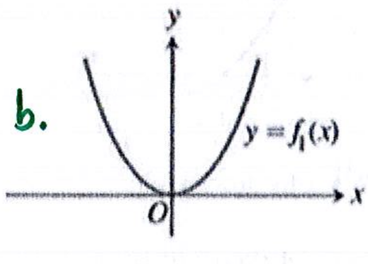
(c)

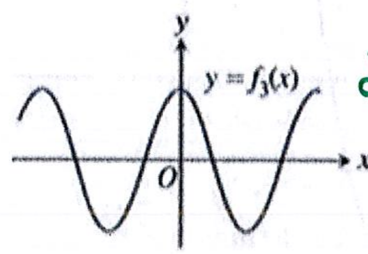
Derivative of $f_3(x)$



(d)

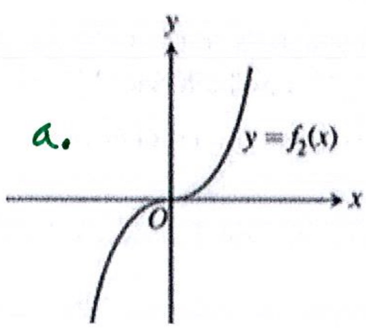
b.

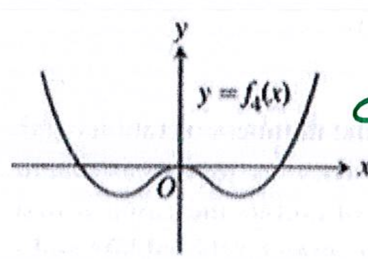




d.

a.





c.

9. If $f(2)=3$ and $f'(2)=5$, write an equation of the tangent line to the graph of $y=f(x)$ at the point where $x=2$.

$y-3=5(x-2)$

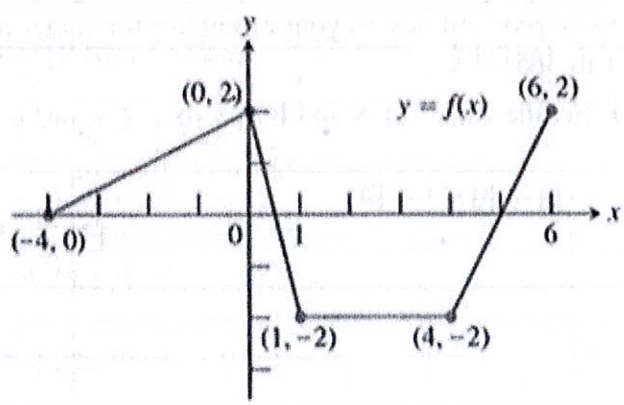
10. If $f(2)=3$ and $f'(2)=5$, write an equation of the normal line to the graph of $y=f(x)$ at the point where $x=2$.

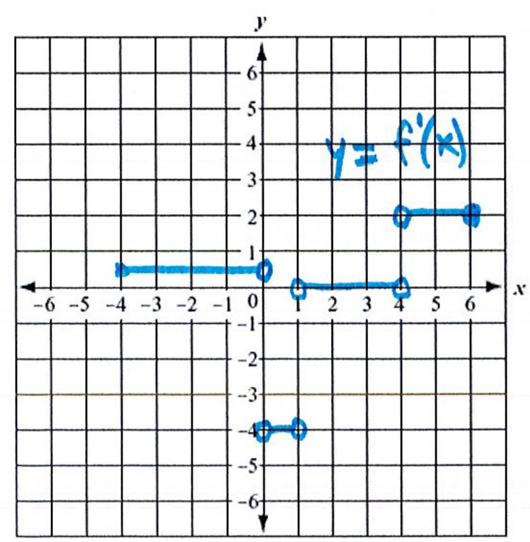
$y-3=\frac{-1}{5}(x-2)$

11. The graph of the function $y=f(x)$ shown here is made of the line segments joined end to end.

a. Graph the function's derivative function.
b. At what values of x between $x=-4$ and $x=6$ is the function not differentiable?

not differentiable at $x=0$, $x=1$ and $x=4$





12. Determine the value(s) of x for which the function $f(x)=|x+3|-4$ is not differentiable.

The vertex of the absolute value graph is $V(-3,-4)$; therefore, the function is not differentiable at $x=-3$

13. Given the function $f(x)=4x-x^2$, determine the numerical derivative of the given function at the point $x=3$ using $h=0.001$.

$$\frac{f(3+0.001)-f(3-0.001)}{3+.001-(3-0.001)} = \frac{4(3.001)-(3.001)^2-(4(2.999)-2.999^2)}{3+.001-3+0.001}$$

$$= \frac{-0.004}{0.002} = -2$$ which is an approximation of the derivative of $f(x)$ at $x=3$, i.e. $f'(3) \approx -2$

14. The derivative function for $f(x)=x^{\frac{1}{3}}$ is $f'(x)=\frac{1}{3}x^{\frac{-2}{3}}$. Is the function $f(x)=x^{\frac{1}{3}}$ differentiable for all elements of its domain? If not, explain why not.

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$ which is undefined at $x=0$, i.e. $f'(0)$ is undefined. $x=0$ is in the domain of $f(x)$ but not in the domain of $f'(x)$.

15. The graph of a function is shown below. State the values of x for which the function is not differentiable.

$f(x)$ is not differentiable at $x=-1$, $x=0$ and $x=2$.